

# Formal fundamentals for the design of manufacturing systems

## Prof. Hermann Kühnle

Head of the Institute for Ergonomics, Manufacturing Systems and Automation, Otto von Guericke University of Magdeburg  
Executive Director of the Fraunhofer Institute of Factory Operation and Automation  
Magdeburg  
Germany  
Universitätsplatz 2  
D-39106 Magdeburg  
Germany  
Phone: +49 391/40 90 100, Fax: +49 391/40 90 102, Email: kuehnle@iff.fhg.de

1. Introduction
2. Tools for the exemplary description of manufacturing systems
  - a) Graph and tensor notations
  - b) Maintaining the integrity
3. Impulses from production technology for research and teaching of the engineering sciences

## 1. Introduction

Even during the emergence of rapidly changing developments of needs and views of life, manufacturing is and remains the source of important knowledge concerning the enhancement of the quality of our life. The answer to the question of the feasibility of the application of technological and technical achievements has been a crucial guide in search of solutions for creating better living conditions for man. Technical products that are items of everyday use gained increasing, even essential, importance for man in the course of this endeavor. The capability of efficiently producing artifacts in large numbers of units is regarded as a dominant orientation feature in assessing the performance of national economies and economic regions.

Many conditions of industrial production have undergone a considerable change recently. Not the technically feasible determines goals and purposes any more, but the economically sensible and the socially justifiable. Thus, not committing oneself to individual technologies is much in demand, but rather optional, situationally conditioned application of technologies, oriented towards market situation and economical boundary conditions. Considering the context with speed objectives that can be registered everywhere, this results in an immediate need of methods and tools for rapid design, rearrangement, and optimization of parts of manufacturing systems or even entire enterprises in total.

With the emergence of further economically efficient nations as competitors of our manufacturing industries, our knowledge of how to produce optimally with regard to economy and technology is being put to the test.

Reviewing the development of different cultures and their respective efficiencies, it may be stated that the top positions of technology repeatedly changed between culture areas in the course of history. In comparative examinations, progress and achievements of mathematics over and over again turn up to be the basis or explanation for increased technical efficiency.

This becomes most striking in the shift of leadership from China to Europe in the course of the 17th century (this finding can be proved quite concretely based on the numbers of inventions). Mathematical work should be mentioned having lasting effects on this development, which has been done by Decartes, Poincaré, Leibniz, Newton as well as Lagrange and Hamilton. Theories and calculi originated that were applicable to technical products. Feasibility studies and dimensioning became possible quickly and without numerous practical experiments. Undoubtedly quite new strengths arose on this basis with quick technological innovations.

Therefore, recalling solid fundamentals to mind that remain generally valid and stable and may be adapted to rapidly changing designing needs, also of manufacturing systems, should be regarded as the fundamental direction of thrust of successful research.<sup>1</sup>

"Diversity that fails to be transformed into unity is confusion;  
Unity that is not based on diversity is tyranny."

Blaise Pascal

With a view to the present situation in which the science of engineering competes with other culture and economic areas, the thesis is often maintained that there should be an even stronger specialization into applications, and that knowledge should be made immediately translatable and applicable as quickly as possible through further increasing practice-oriented knowledge. In university courses of training, in which the engineering sciences are ever increasingly focusing on application (often to the disadvantage of fundamentals), the duration of studies keeps getting considerably longer. In many cases, another prolongation of studies is implied in offering so-called supplementary courses (i.e. after the end of the study).

The subsequent considerations are elaborated following system science approaches according to which models and real systems always group around a formal theoretical kernel.

The interlinking of this theoretical kernel with models and the real systems covered by them leads to phenomenological laws that can be proved both inductively from empirical

---

<sup>1</sup> In this, the term 'manufacturing system' should be seen as approximately synonymous with the term 'manufacturing plant'.

findings of real manufacturing systems and pertinent model statements (inwards), and deductively in an analytical way from the theoretical formal kernel (outwards) (Fig. 1).

This relation is taken up as an example, for possible extension for formal fundamentals and impulses for a trend-setting rearrangement of university teaching towards manufacturing will show.

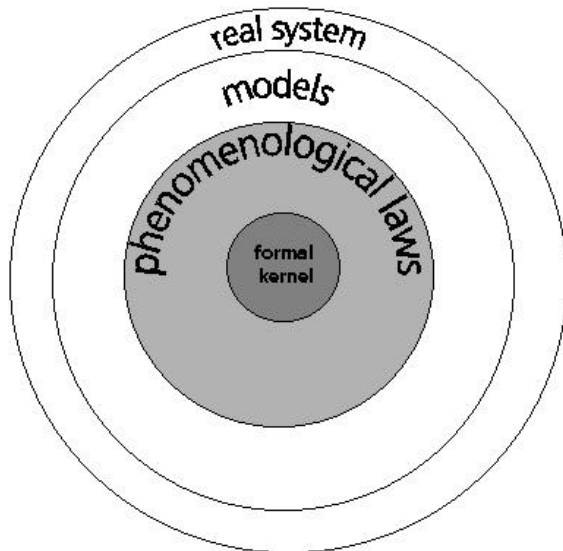


Fig. 1 Shell structure of system sciences [Kornwachs]

## 2. Exemplary description tools for manufacturing systems

The objects of investigation getting increasingly more complex, as well as the growing need of formal description possibilities, which are also adequate to information processing possibilities, have given rise to a number of separately created approaches, models, description tools as well as EDP-supported systems.

A common theoretical basis, an integrative formal, generally valid kernel comprising neutral descriptive elements is overdue. It will allow and accelerate the elaboration of systematizations, classifications and interlinking of individual models and promotes the concentrated advancement of additional fundamentals.

Regarding the development of system sciences, the shell structure mentioned may be verified concretely for the examples taken from the engineering industry as well as from branches of production engineering [Kornwachs, Bjørke]. Consistently continuing the reasoning lying at the bottom of it and further filling in the formal kernel immediately results in important needs of extension. This will be exemplarily proved in the following.

### 2a) Graph and tensor notations

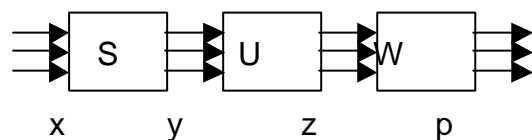
Production as being any value-creating combination of resources is a transformation that, based on energy, information and materials, brings forth products. This connection can be formulated as the relation  $S$  with input  $x \in X$  and output  $y \in Y$  with  $xSy$  [Pichler]. To begin with, one-stage correlations will be discussed where  $S$  is the production function  $S : X \rightarrow Y$ .

This case is also penetrated using efficient graph-theoretical descriptions as a "goes into connection", which appear as the formal kernels of all PPS software (Fig. 2).

$x \in \mathbb{R}^m$  and  $y \in \mathbb{R}^n$  result in the immediate integration into vector space theory, particularly the theory of the systems of linear vector spaces [Schaal].

Because of  $\{s_{ij}\} \in S$  and  $s_{ij} \in \mathbb{R}$ , the rows of the linear production function  $S \rightarrow y = S \cdot x$ , which can be represented as a matrix, may also be interpreted as vectors  $s_i$  in  $\mathbb{R}^n$ , related to a dual base, thus also as contravariant vectors (from the orthogonal space). This results in the immediate possibility to introduce tensor notation  $y_i = s_i^j \cdot x_j$ .

This notation facilitates particularly the representation of the link-up of transformations, which are so important for the consideration of production, as follows:



$$y_i = s_i^j x_j \qquad p_q = c_q^j x_j$$

with  $c_q^j = v_q^l u_l^i s_i^j$

[because of  $p_q = v_q^l z_l = v_q^l u_l^i y_i = v_q^l u_l^i s_i^j x_j$ ]

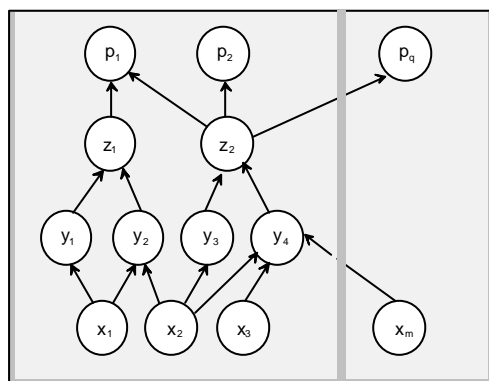


Fig. 2 Gozintograph of an event tree

$x_j$  with  $j = 1 \dots m$  input manufacturing system (materials, energy,...)

$y_i$  with  $i = 1 \dots n$  output 1<sup>st</sup> production stage = input 2<sup>nd</sup> production stage

$z_l$  with  $l = 1 \dots o$  output 2<sup>nd</sup> production stage = input f<sup>th</sup> production stage

$p_q$  with  $q = 1 \dots r$  output  $f^{\text{th}}$  production stage = output manufacturing system with  $f$  - number of production stages

For dynamic considerations, mappings of transformation fluxes, consequently of input/output rates, are to be used. The introduction of the "natural time" as the time set  $T$  and interpretation of the above equations as a set proposition integrated over a time period results in the fluid-flow equations as the 1<sup>st</sup> time derivative:

$$\dot{p}_q = \dot{c}_q^j x_j + c_q^j \dot{x}_j = c_q^j \dot{x}_j$$

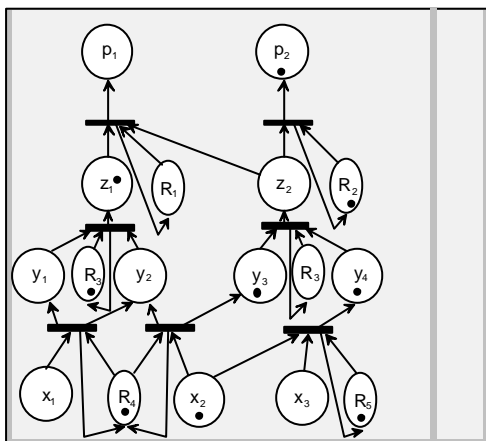
(as  $c_q^j$  is constant).

The transition to the discrete time set ( $\cong N_0$ ) leads, to integrals, used as a basis in *MPR-Systems*.

$$p(n) - p(0) = \int_{n-1}^n \dot{p} dt + \dots + \int_0^1 \dot{p} dt$$

$$= p_n + p_{n-1} + \dots + p_1$$

The extension of views to flow notation as well as the consideration of resources leads to the *Petri-Net* as an extension of the gozintograph. That comprises not only the set of vertices  $E: X \cup Y \cup Z \cup \dots \cup P$  (input and output elements) and the edges  $V: S \cup U \cup \dots \cup W$  (connections), but also sites  $R$  (resources) containing machine and plant states along with the allocation of effecting activities/processes (transitions). The flows of sets are expressed in units moved (tokens).



$x_j : y_i : z_i ; p_q$  as in Fig. 2

$R_k$  with  $k = 1 \dots u$  resources of the manufacturing system

Fig 3. *Petri-Net-Model* of a manufacturing system

Besides the "goes into graph", another homomorphic mapping can be obtained from the *Petri-Net*, the directed graph of transformations and transformation progress. The representation as a system has again a close connection.

Referring to inputs (X) and outputs (Y), this is based on the so-called Black-box approach.

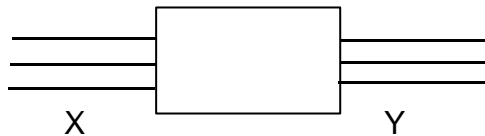


Fig. 4 Production as I/O - System

A more detailed description uses the state mapping, which extends the Black Box to state parametrization  $(x, y, Z, \lambda)$ .

$(x, y, Z, \lambda)$  is the state parametrization for  $(x,y,S)$  with state set  $Z$  and state parametrization mapping  $\lambda$ , which assigns different levels depending on inputs and outputs to the system states  $Z$ . [Pichler]

$$\lambda: Z \Rightarrow P(X \times Y)$$

P: Set of  $X \times Y$

Based on this extension, numerous possibilities for differentiating arise depending on whether the state parametrization breaks up inputs or outputs.

With regard to the manufacturing system, this means that certain inputs generate only certain outputs, which do not develop from other inputs outside the inputs partial set considered. In this case, the state parametrization would break up both inputs and outputs. For structuring decisions on manufacturing systems, this would have to result in considerations concerning the separation of the manufacturing system into two units.

Besides set connections between input and output, additional quantities may be selected and compared. This includes *time periods* taken from the *time set*  $T$  as well as comparisons relating to the basic equation  $V(x) < V(y)$  or  $V(y) - V(x) > 0$ , i.e. the value of the object produced exceeds the value of the resources and materials.

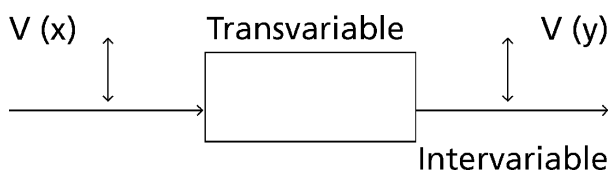


Fig. 5 Variables of I/O - System

Due to the different natures of these quantities, the measurement comparison quantities obtained across a system element are described as "transvariables". Opposed to that, the inputs and outputs connecting elements are characterized as "intervariables".

With reference to tensor notation, transvariables may be expressed in contravariant notation.

**Example:**

A productive unit is represented as real-valued intervariable and transvariable in the notation given. An especially versatile field of application for this notation, which is commonly used in enterprise practice, is given by Wiendahl (Fig. 6).

The intervariables mentioned are accumulated there, separately for inputs and outputs, across all units considered during the time period  $t_2 - t_1$ , not as item opportunities, but as time consumption rates.

$$\frac{h}{E_j} x_i^j \text{ or } \sum_i \frac{h}{E_j} \int_{t_1}^{t_2} x_i^j dt$$

The comparison with the corresponding sum on the output side allows to generate additional transvariables, such as in quantities differences, time differences etc. as well as representing and comparing with transvariables, and resulting characteristic functions.

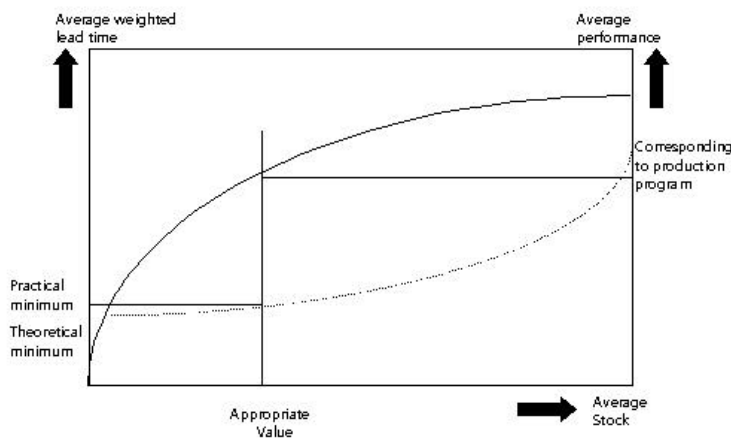


Fig. 6 Operation diagramm [Wiendahl] Transvariables over Intervariable [Stock]

Going back to the starting point of *Petri-Net-Models*, the projection of covariant (inter-variable) quantities on the one hand and the contravariant (transvariable) quantities on the other hand leads to homomorphic mappings of the graph with the following assignments:

covariant (intervariable)

Relations between transformations are ascertained. These entail set consideration options and calculations of transit times between transformation steps as well as material flow dimensioning. The pertaining graph is used in practice as a presentation of connections between activities.

contravariant (transvariable)

It is transformation that is focused on. The resulting graph allows statements with respect

to time, load evaluations etc. An example for applied graphs is the representation of the Critical Path method (CPM).

As is easily imaginable, in the first case, vertices take over the role of edges in the second case. In other words, the assignments transvariable and intervariable/contravariant and covariant are exchanged once you change over to the opposite principle while maintaining topological properties (free, directed, connected etc.).

This is an example for a well known basic principle, the duality principle, which reads simplified: Statements of the covariant "world" remain valid in the contravariant "world".

This principle is reflected in a fundamental law in production sciences: Structures and sequences, determined by quantity rates on the one hand and diversity on the other hand, have to be selected quite differently. The quantity-related deepening is done through the contravariant (transvariable) description, whereas the consideration responding to diversity is covariantly (intervariably) stressed.

A phenomenological law in production confirms: Structural solutions are always strictly alternative, either seen from a quantity point of view or based on diversity requirements — yet never appropriate in mixed form [Warnecke]. As is generally known, structural and sequential design based on quantities/repetition rate has led to efficient approaches and methods for functional differentiation and optimization. The sudden change to diversity and rate requirements faced the manufacturing systems so designed with almost insoluble problems. It was evident that the change to the "duality principle" was hard to realize although there is a close problem/system alliance. Therefore, there is a close relation despite a fundamental dissimilarity. A consecutive differentiation can be done using the moving space model, which connects both principles on a superordinate level, the organizational level of the manufacturing system.

Using analysis and position determinations, the law is applied according to quantity and diversity for the running manufacturing operation for immediate structural adaptation (structural partitioning or integration). The underlying mechanisms of activity are those of interconnected control systems.

The thus inserted superordinate level may be introduced on the *Petri-Net*, which results in the extension of the *Petri-Net* by information and decision modules.



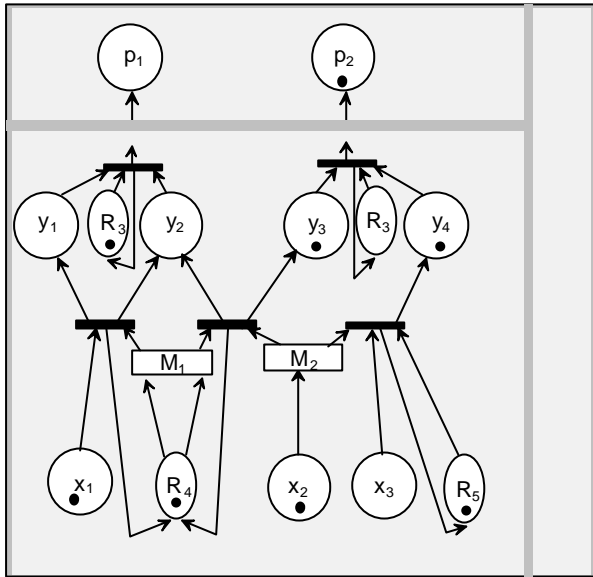


Fig. 7: Petri-Netz-Extension to *Agent system*

$x_i$ ;  $y_i$ ;  $z_i$ ;  $p_q$ ;  $R_k$  as in Fig. 3  
 $M_s$  with  $s = 1 \dots v$  methods

## 2b) Maintenance of holism

The structure of the formal description using transvariables and intervariables can be further generalized. Particularly the complexity of the manufacturing plant becomes thus formally perceptible. This complexity often led to hints concerning the interdisciplinary nature of the object of investigation 'manufacturing system'. For instance, [Spur] states economics, social science, information science and others.

For the manufacturing system as the scientific object 'factory operation' in a narrower sense, such a classification can have problematic consequences. The scientific disciplines that are applied in a bundle or from time to time developed independently without considering the other disciplines also mentioned and, most of all, without the specific object background. Thus, these are pure science complexes that can be applied even without factory-operational overall connections. Above all, the science disciplines mentioned will keep up their independence and only slowly integrate the developments of technology acting on them. Another problem may arise from the autonomous, different notion of science that the science disciplines are based on.

To face the mentioned and additional difficulties from the outset, a different approach is preferred for production sciences. A formal mathematical kernel is to be used as the basis for factory operation. This initially appears to be a withdrawal to formal fundamentals whose relevance for the applications seems to be little. The diverse possibilities resulting from using this formal kernel warrant this approach.

For dealing with aspects due to the complexity of the object of investigation, a consideration of mutually independent partial systems is proposed that are derived from graph and system formalisms. Both intervariables and transvariables are obtained from measuring values, defaults of quantities to be achieved or maintained, and attributes as well as assignment provisions. This results in an extension to form a layer model integrating these aspects [Kühnle] (Fig. 8).

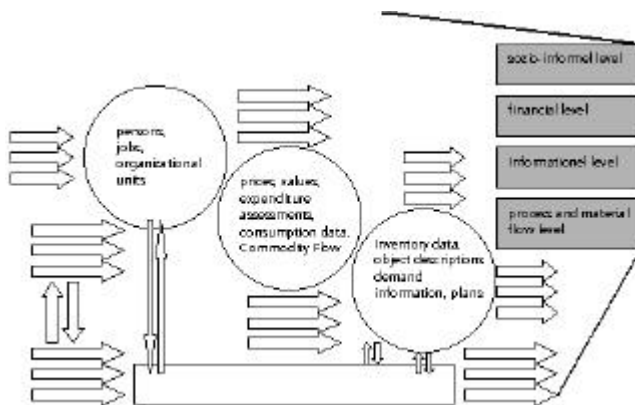


Fig. 8 Layer model for the aspect-related oriented discussion of manufacturing systems

### 3. Impulses from production engineering for research and the teaching of manufacturing sciences

Due to ever increasing technical possibilities and capabilities, the production sciences are subject to high development speed. At the same time, there are intricate expectations with regard to quickly applicable new solutions in all fields of application. Often, earlier specialization in multifarious applications is demanded as a structural measure in training, science, and research. Advantages are made out in shorter training time, immediate applicability of training and research contents as well as meeting the requirements created by the diversification of applications. This requirement results in considerable endeavors towards the establishment of additional research disciplines and courses of studies at universities. It begins to show that this line cannot be followed for any length of time.

International comparability claims and the warranty of qualitative standards clearly reveal the limits. In this paper, therefore, the enormous possibilities have been pointed to that are open to the further development of research and teaching in the field of production sciences through consistently widening and consolidating the theoretical fundamentals. Then, there is the need for consistently remembering the formal disciplines. It is possible and appropriate, in an integrative and generally valid way using formal notations and elements, to adapt generalized system theories for many engineering subject matters.

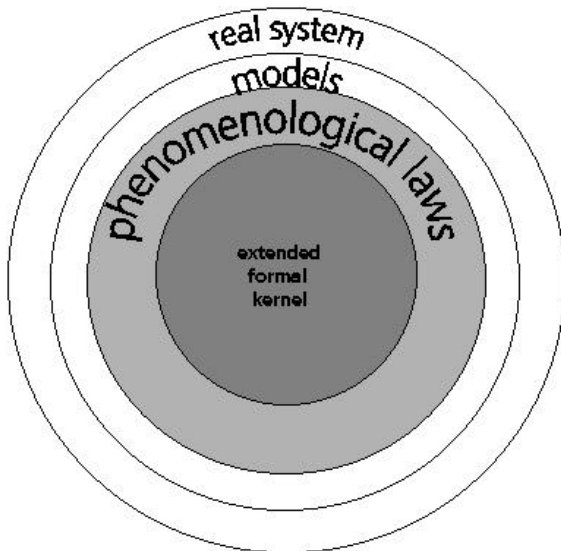


Fig. 9: Shell structure comprising an extended formal kernel as a basis of production operational research

Unfortunately, this is still realized today on a narrower common theoretical foundation such that independent notations and definitions are created for each specializing discipline. This implies for engineering students at universities that the very same subject matter has to be multiply imparted to them with varying looks (labels) [Bjørke]. In order to stop this development and to reverse the tendencies with all their negative consequences, extended fundamentals should be made available (Fig. 9).

The formal basis of training can be created using geometrical, measure-theoretical and function-theoretical foundations kept homogeneously, based on which further specifications can be carried on into single-field connections up to applications. In this way, the generally perceived theory deficit might be covered, on the one hand, and the accuracy of training increased, on the other hand. Simultaneously, the period of training would be substantially reduced while increasing the diversity of applicability in various fields and disciplines.

Production engineering as a consequent system science has always proved up to now that, due to applicative necessities as well as technological innovations, it exerts an influence on almost all sciences and opens up new fields of application. It is, therefore, highly fit to realize the consistent widening of the theoretical foundation.

It should be particularly indicated at this point that production engineering/production sciences always end up closely interlaced with the basic disciplines of natural sciences and mathematics. The associated industries (engineering industry, processing industries, automobile industry, aircraft industry etc.) have belonged, now as before, to the most knowledge-intensive and innovative branches ever. Thus, work done towards formalization is to be interlaced most closely with the resulting applications in order to be able to quickly gain lasting competitive advantages from the shift of emphasis in knowledge presentation and preparation.

## References

BjØrke, Ø.: Manufacturing Systems Theory, Tapir Publishers 1995

Hartmann, M.: Merkmale zur Wandlungsfähigkeit von Produktionssystemen bei turbulenten Aufgaben; Stuttgart Logis, 1995

Kornwachs, K.: Vom Naturgesetz zur technologischen Regel - Ein Beitrag zur Theorie der Technik

In: Banse, G.; Friedrich, K.: (Hrsg.) Technik zwischen Erkenntnis und Gestaltung. Berlin 1996

Kühnle, H.: u.a.: Produzieren im turbulenten Umfeld In: Warnecke, H.-J.: (Hrsg.) Aufbruch zum Fraktalen Unternehmen. Berlin, New York, 1995

Pichler, F.: Mathematische Systemtheorie - Dynamische Konstruktion; Walter de Gruyter, Berlin, New York 1975

Schaal, H.: Analytische Geometrie 1-3, Stuttgart 1976

Spur, G.: Fabrikbetrieb : das System, Planung, Steuerung, Organisation, Information, Qualitaet ; die Menschen; München, Hanser 1994

Warnecke, H. J.: Revolution der Unternehmenskultur: Das Fraktale Unternehmen, 2 Auflage, Berlin, Heidelberg, New York, London, Paris, Tokyo, Springer 1993

Wiendahl, H.-P.: Nyhius, Peter: Engpaßorientierte Logistikanalyse; München, Transferzentrum 1998